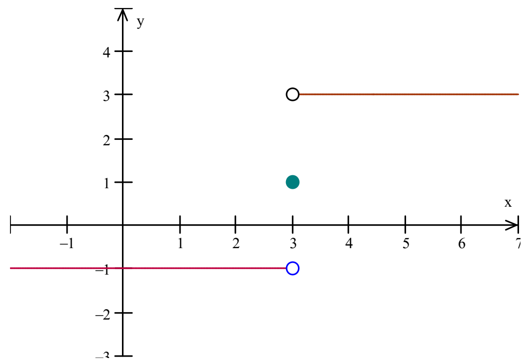
2.1 ….some comments

1)



a) $\lim_{x\to 3^-} f(x) =$

What is a limit?

A limit is a y-value – the one that is being approached by the y-values that are related to x-values getting closer and closer to the x-value of interest (in this case, 3).

What is the superscript '-' after the 3?

This tells us that, as we consider what the limit is, we are only interested in the x-values to the left (the negative side) of 3.

The negative side?

As we look at a 'normal' horizontal number line, the negative numbers are on the left, positive numbers on the right. So … it's just a "mathy" way of saying "left"

I know $x \to 3^-$ means 'approaching 3 from the left' … but where do I start?

Ah – excellent question … and a holdover from the ideas we have in English about what the words mean.

To step back slightly: $f(3)$ *refers to the value of the function f (a y-value) when the x-value is 3.*

$\lim_{x\to 3^-} f(x) =$ … *refers to a y-value that the values of the function f are "very near" when the x-values we are considering are "very near" x=3 (and in this case, only on the left). In fact, the x-values have to be near 3 and not include 3!*

The other thing to remember is that we are talking about a function. So let's get a crazy function analogy going (or is this a metaphor?). Assume you are in a car that travels left and right along the x-axis. Tied to the bumper of your car is a string with a balloon on it. And as you drive along, the height of the balloon changes ….. <magic!> The height of the balloon will be given by our function f – got the picture? The position of the car (the x-value) and the height of the balloon (the y-value) are tied together by the function f.

Now …. Start driving the car at x=0. Drive east (RIGHT). Nothing matters until you get close to 3. As you get closer to 3, pay attention to the height of the balloon. If you could go one POINT further and actually drive ONTO 3 …. What would the height of the balloon be?

That's the limit.

So … the limit 'idea' is really talking about the x-values ….. the y-values only enter in because they help supply the "answer".

Anything else I can look at?

I think http://www.calculus-help.com is the best introduction to this new idea. Click on the Limit1 and Limit2 videos …

OK … so what is the answer to 1a) ?

Since the function f only has the value -1 for EVERY SINGLE x-value to the left of 3 … the limit is -1.

$$\lim_{x \to 3^-} f(x) = -1$$

1b) $\lim_{x \to 3^+} f(x) = 3$. *Similar reasoning allows us to see that f(x) is only ever 3 when x is to the right of x=3 ….*

1c) $\lim_{x \to 3} f(x) = ?$ When there is no 'superscript –' or 'superscript +' on the *x*-value we are approaching, then the problem asks for a **two-sided** limit. Hardly any thinking involved !! What is the left hand limit? -1! What is the right hand limit? 3! Are they the same? NO! So the limit **DOES NOT EXIST**! We abbreviate this DNE ….

1d) $f(3) = 1$. Just the value of the function.

1e) $\lim_{x \to -\infty} f(x) = ?$

Hmm … there is no number we are 'approaching' … only negative infinity. I thought negative infinity was NOT a number?

Good observation! And so this is a specialized shorthand. In fact all math symbology is a kind of shorthand to avoid saying the same phrases over and over again.

So how does one "approach" infinity? By considering x's that are MORE and MORE negative.

And what does this mean?

This limit statement just asks for the behavior or pattern in the y-value of a function as we consider more and more negative values of x.

Hey!! Isn't this the same as the "**end behavior**" we talked about in Precalculus?

YES!

This limit is -1. $\lim_{x \to -\infty} f(x) = -1$.

1f) $\lim_{x \to \infty} f(x) = 3$.

Note that limits "**to infinity**" like these last two are ALWAYS one-sided limits!

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2)

a) $\lim_{x \to 2^-} f(x) = 2$

b) $\lim_{x \to 2^+} f(x) = 0$

c) $\lim_{x \to 2} f(x) = DNE$

d) $f(2) = 2$

e) $\lim_{x \to -\infty} f(x) = 0$

f) $\lim_{x \to +\infty} f(x) = 2$

4)

a), b), c) $\lim_{x\to 0^-} g(x) = \lim_{x\to 0^+} g(x) = \lim_{x\to 0} g(x) = 3$

d) $g(0) = 3$

e) $\lim_{x\to -\infty} g(x) = \infty$

f) $\lim_{x\to +\infty} g(x) = \infty$

6)

a) $\lim_{x\to 3^-} F(x) = 2$

b) $\lim_{x\to 3^+} F(x) = 2$

c) $\lim_{x\to 3} F(x) = 2$

d) $F(3) = 3$

e) $\lim_{x\to -\infty} F(x) = -\infty$

f) $\lim_{x\to \infty} F(x) = \infty$

15) and 16) are important … At least from the perspective that you can easily demonstrate an understanding of limit ideas by applying them to create graphs and functions. So try these!!

19)
a) I'm pretty sure I don't like these questions, but they can help one see what is going on. This is **like** the 'discovery phase' the book talks about. You are asked to guess what $\lim_{x\to 1}\frac{x-1}{x^3-1}$ is by substituting values close to $x = 1$ into $\frac{x-1}{x^3-1}$.

First potential calculator lesson –
There's actually a fairly easy way to do this. Go to the `Y=` screen, and set `Y1=(x-1)/(x^3-1)`. Now `2ND QUIT`. On the home screen, you'll type `VARS`, then > to `Y-VARS`, then hit `ENTER` or `1` (to select `FUNCTION`), and finally `1` for `Y1`.

Now, on the clear screen, you have a `Y1`. Next to it – right where the cursor is located – type `(2)`. You now have `Y1(2)`. Hit `ENTER` – you have just evaluated whatever expression is in `Y1` at the x value of 2. It works exactly like a function reference!

The lesson is not over. Instead of going through all that again for the next value we need to substitute in (1.5), simply type `2ND ENTER` – it brings the previous command back – BUT IT ALLOWS YOU TO EDIT IT! So backspace over the 2 and type 1.5 and hit `ENTER` again. Continue through all the values the problem asked you to substitute in.

End of calculator lesson ☺

You'll get something like:

(for numbers above 1): .142857, .210526, .302115, .330022, and .333000
(for numbers less than 1): 1, .571429, .369004, .336689, .333667

This exercise is supposed to get you to see that, as the x-values get closer to 1, the y-values – the values of the expression – get closer and closer to *something*. What does it look like? If we average .333000 and .333667 we seem to get 1/3 – and so that will be our guess of this limit.

Go ahead and graph the expression, and see if you also get something close to a third.

Now – what would the verification step look like? We could try substituting $x = 1$ into the expression $\frac{x-1}{x^3-1}$. Then we get 0/0; an indeterminate form (it's called an indeterminate form because it could be ANY number: 0 * 1 = 0; 0 * 2 = 0; 0 * 3 = 0; and so on ….) Anyway – we have to try something else.

If you "remember" that $x^3-1=(x-1)(x^2+x+1)$ [don't be alarmed if you think you never saw that before!] then an algebraically equivalent limit is:

$$\lim_{x\to 1}\frac{x-1}{x^3-1}=\lim_{x\to 1}\frac{x-1}{(x-1)(x^2+x+1)}=\lim_{x\to 1}\frac{1}{(x^2+x+1)}=\frac{1}{3}$$ (by substituting in the 1).

But that's for another lesson!

25) Tackling 25 on the calculator is pretty much the same thing, but to substitute values in and guess a limit (if one exists) as x gets larger and larger you need to use larger and larger numbers. I just keep adding 0s … until the calculator can't handle any more.

… and more discussion in class !!